**Green’s with holes & more**

Assumption: All curves C below are assumed to be 1) closed 2) simple 3) counter clockwise 4) traced once; and 0) C is smooth (except for finitely many points) (like a rectangle)



The main feature of this specific line integral is that Nx –My=0 away from the origin. That is,  except at the origin.

**Answer:** i) If C is around the origin, **Answer = 2π.** Idea of proof: Here you need Green’s with holes to “simplify” C into a circle C’ (inside C) around the origin by using equation (\*). Then you either parametrize the circle Or use the half-substitution trick to make Green’s Thm. Applicable

ii) If C is NOT around the origin, **Answer = 0 (zero).**

2) A related line integral is



**Answer**: i) If C is around the origin, **Answer** = **200π + 5 Area R** (where R is the region inside C).

The easiest way to see it, break the line integral into 2 parts.

ii) If C is NOT around the origin, **Answer =** **0 + 5 AreaR.**

**3) A Challenging Question:**

We know that the famous Gauss differential form 

satisfies the condition Nx=My away from the origin (where M=coef.(dx) & N=coef. (dy)).

**Find a potential function** of our differential form on the region R where

i) R is the right-half plane (x>0) (which is a simply connected region)

ii) R is the left-half plane(x<0) (which is a simply connected region)

iii) R is the upper-half plane (y>0) (which is a simply connected region)

(i++) Compute the line integral of our form from A(2,2) toB (7,5) if C lies in the right-half-plane

**Answers (i):** f=. Proof: By inspection, we try f=. Then both are correct!

(ii) Also  (iii)  (i++) 

*Good Luck*